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VIRTUAL SCIENCE

VIRTUALITY AND KNOWLEDGE ACQUISITION IN SCIENCE AND COGNITION

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The focus of this paper is the process of knowledge acquisition (KA) and which role virtuality plays in this context. We argue that there are three different modes of knowledge acquisition which can be identified both in the domains of cognition and science: the empirical, the "constructive", and the "synthetic" mode. We show that the method of constructing knowledge in the virtual domain (i.e., the synthetic mode of KA) is not only a principal mode of KA in our cognition (e.g., thought experiments, making plans, etc.). It becomes increasingly important in the field of (natural) science in the form of simulations and virtual experiments. The attempt to find an answer to the question of whether simulation can be an information source for science, and to validate the computational approach in science, leads to a new interpretation of the nature of virtual models. This new perspective renders the problem of "feature extraction" obsolete.

1. Introduction

What are the main characteristic features of a cognitive system? Above all, it is a living system. As such, it has to be capable of (i) *acquiring knowledge* from the environment and of (ii) *representing* this knowledge about the environment in one way or another. Research in cognitive science has developed highly sophisticated (as well as highly divergent) concepts of knowledge representation (see e.g., Bechtel et al. 1998, Brook et al. 2000, etc.). However, we will not focus on the second question of how knowledge is represented in cognitive systems. Rather, the following question will be addressed: How can knowledge be *acquired* by interacting with the environment? And, how can knowledge about this environment be *constructed* on the basis of these "empirical findings"? More specifically, which *modes of knowledge generation* and *knowledge acquisition* (KA) can be found both in the context of cognitive processes and of theory development in science? As has been shown in Peschl (2001) there are close relationships and structural similarities between the process of science and cognition—in this paper, the focus will be on the process of knowledge acquisition and production in cognition and science. We are going to do that from an epistemological as well as from a philosophy of science perspective.

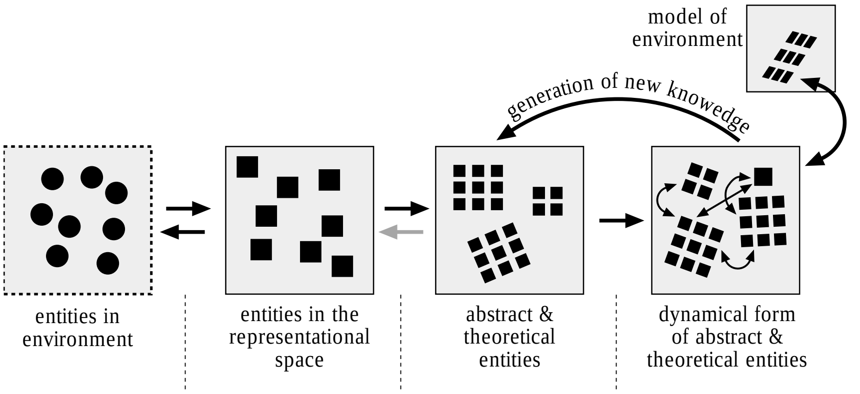
In the course of this paper, it will be shown that three different modes of knowledge acquisition can be identified both in the domain of cognition and of science: (a) the empirical mode, (b) the "constructive" mode, and (c) the "synthetic" mode of KA (see section 2). It will turn out that the concept of *virtuality* plays an important role in this context. Section 4 will (i) develop the implications of the "virtual mode" of knowledge acquisition for the process of science and cognition; and (ii) investigate the nature of virtual models. Finally we will examine how the virtual approach extends the traditional scientific empirical method by introducing the concept of *simulation* in the process of theory formation.

2. The Three Modes of Knowledge Acquisition in Cognition and Science

In order to survive, every organism necessarily needs some kind of knowledge/representation[1] of its environment. Such a statement is in accordance with the standard definition of an *anticipatory system* as presented by Robert Rosen (1985): It is a system "containing a predictive model of itself and/or of its environment, which allows it to change state at an instant in accord with the model's predictions pertaining to a latter instant" (p. 339). In this paper, the question of interest is, how this knowledge is acquired from the environment. Our focus is on cognitive processes in higher animals as well as in science. In addition to the similarities between scientific and cognitive processes developed in Peschl (2001), it will be shown that there exist significant parallels regarding the processes of KA as well.

Here is the basic situation which we are confronted with when we are investigating the representational capabilities of an organism. On the one hand, there is an organism which has to generate adequate behavior for its survival. In order to do so, it is obliged to acquire and make use of knowledge about the structure of its environ-

1. A rather *broad notion* of the concept of "representation" is used here. It ranges from very simple forms of representation in primitive organisms, such as the embodiment of knowledge in physical or biochemical structures (e.g., the knowledge enabling an organism to follow a chemical gradient), "up to" complex symbolic or logical structures describing the world.



	(i) empirical mode	(ii) constructive mode	(iii) synthetic mode
domain of cognition	perception (& action)	generalization, induction & abduction	thought experiments making plans imagination
domain of science	process of measuring (experiment)	induction & theory construction	simulation in the virtual domain

Figure 1: Three modes of knowledge acquisition and construction in (human) cognition and the process of science.

ment. On the other hand there is the surrounding environment having a certain structure and dynamics which is only partially known[2] by this organism.

For an observer, the question arises: How can the—for the particular system—“relevant” features of this environment be isolated and obtained? How is the knowledge generated which is necessary for the organism’s survival? It seems that there is some kind of transformation going on within the organism that converts those relevant features into knowledge structures which enable the organism to generate adequate behavior?[3] Observing our own cognitive abilities as well as the process of science one can discover that there exists more than one mode of knowledge acquisition. Figure 1 graphically shows three different modes of KA: (i) the empirical mode, (ii) the mode of construction and abstraction, and (iii) the “virtual mode”.

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2. To answer the question whether knowing an environment only partly (or not at all) refers to a fundamental impossibility—like “(for epistemological reasons) it can never be known” or is it not yet known—would transcend the scope of this paper. However, as we will show in section 4, this epistemological question may render superfluous for very different reasons.
3. Of course, this question touches the whole problem of *learning* in representational systems. However, we are *not* going to discuss this problem, but we will take a more global look at the epistemological modes of knowledge acquisition.

2.1 The "Empirical Mode" of Knowledge Acquisition

The classical mode of knowledge acquisition is what can be referred to as the "*empirical approach*". Here, a certain aspect of the environment is detected or perceived by a sensory system. In a (natural) cognitive system this is assumed to be realized in the process of perception in the sensory system and in the first steps of processing these incoming signals. In the context of science, this mode of knowledge acquisition is the most basic and classical approach to any environment: namely, to make an experiment and to use gauges for detecting certain environmental states.

What is happening in this mode of KA from an epistemological perspective? The sensor system is sensitive to a certain state or aspect of the environmental dynamics. Whenever this state occurs in the environment and the sensory system is present at the location of this state (transition), a change of states is triggered in the sensory system as well, i.e., the environmental signal is transformed into an "internal signal"; in cognitive systems this process is referred to as the process of *transduction*. Keep in mind that due to the internal dynamics of the sensory system, its states do not exclusively depend on the environmental states, but also by its own *history of state transitions* and, thus, by its current internal state. Similarly, Foerster (1982) distinguishes between trivial and non-trivial machines. It is impossible to fully describe the latter, since its internal states depend on the history of the system rather than exclusively on the current input. As an example think of the highly adaptive processes in our visual or auditory sensory systems: Due to external inputs the internal states of the sensory changes and becomes less receptive to the input. From an outside perspective this can be interpreted as "adaptive behavior". From an internal perspective this means that the environmental input is distorted.

What is the result of these transduction processes? What kind of knowledge can we expect to be "acquired" from the environment by applying this mode of knowledge acquisition? Both in science and in cognition this process of perception results in some kind of *primary representation* of the environment (i.e., primary signals with representational function). This means that basic features or states of the environmental structure and dynamics are represented as certain states in the sensory system and in the primary representational processes. However, from what has been said above follows that these representational states are *not* some kind of direct mapping of the environmental state—rather they are *system-relative* (in the sense of theory-laden) states which are modulated by the environmental dynamics (e.g., Riegler, Peschl & von Stein 1999). One can think of this primary representation as a rather unordered collection of data which are (a) referring indirectly, i.e., via the constructive transformation process of measuring or transduction[4], to certain environmental states and (b) are coded in the system specific code of the representational system, be it in patterns of neural activations or electrical charges in a gauge.

From an epistemological perspective, there is a fundamental/ontological abyss between the original environmental states and the representational states—the gap between these domains is bridged by the process of transduction/measuring. In any case, this “primary knowledge”[5] represents the point of departure and the foundation for every other operation in the representational domain.

2.2 The “Constructive Mode” of Knowledge Acquisition: Abstraction, Induction, and Construction

In most cases the system-relative primary representational states are per se rather worthless, because (a) they represent only a certain environmental state at a specific moment in time and at a specific place in space and, therefore, (b) are an unordered and uncorrelated collection of representational states or “data” (even if they have been collected with a specific purpose, they are just a set of data which will be brought into some order only if a process of interpretation occurs). However, what is of interest for generating adequate behavior or a scientific theory, is an answer to the question whether a kind of *structure* is present in these data: classes, correlations (within one modality and/or more modalities), trends and patterns (in time and space), etc. Hence, it does not suffice to just collect data by exposing the (natural and/or artificial) sensory systems to the environment, but to process these data/signals in such a way that they provide a basis for generating adequate behavior—cf. the example of Kepler in section 4.3.

Active constructive processes of *abstraction*, *classification*, *induction*, and *abduction* are necessary for bringing some structure into this unordered set of data. By providing a theoretical context—be it a scientific theoretical framework or already existing common sense knowledge structures—the semantically neutral values/signals are brought into a semantic context. This way, a semantic value or meaning is induced into the data. Furthermore, active processes of (re-)arranging, searching structures and regularities, etc. are applied to these data in order to induce and/or project some (again system-relative) classification, spatial, and/or temporal structure and order into/onto the formerly unordered and uncorrelated set of signals/data. These active processes of construction refer to the huge body of questions concerning the problems of learning, induction, classification, adaptation, evolution, etc. and have been discussed in great detail elsewhere, e.g., Holland et al. (1992).

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4. This process introduces a historical aspect which will become an important issue for arguments made later in section 4.3 when we discuss purely logical deductions within virtual simulations.
5. Compared to scientific theories or our own (common sense) knowledge about the world or other people, this “primary knowledge” appears rather primitive.

As opposed to "empirical knowledge", the constructive mode results in knowledge structures which can be characterized in the following way:

- The *epistemological relationship* to the environmental structures is a relationship of functional fitness in the sense of Glasersfeld (1984, 1995).
- It is not knowledge about a specific entity/state in the environment, but it has "*universal/general*" character (within a certain domain). Hence, it is knowledge which represents some general temporal and spatial *regularities* or patterns among entities of the primary representation. In this respect the constructive as well as the system-relative character of the knowledge becomes especially evident.
- In general, knowledge does not only capture a static states of the environment, but also describes its *temporal dynamics* (e.g., in differential equations, in recurrent neural architectures, etc.) This makes it especially interesting for predicting certain phenomena.
- In many cases, this knowledge is not coded in some purely qualitative and/or subjective terms, but is an *operational knowledge*, i.e., knowledge, on which operations can be carried out and which enables us to carry out actions.
- One of the implications of the operational character of knowledge is that one can make *predictions* by applying operations on this knowledge. For example, by setting the variables of an equation to concrete values it is possible to derive a concrete result by applying the mathematical operations which are necessary for solving the equation. This result can be interpreted as a prediction which follows from the general operational knowledge combined with the concrete situation (being represented by concrete values in the variables). Predictions are not necessarily restricted to applying deductive or purely logical methods (an alternative would be the use of, say, analogies)—section 4.3 will further develop this issue.
- From prediction one can go one step further to the *manipulation* and *control* of environmental structures and dynamics, i.e., in many cases the general knowledge about the internal mechanisms can be applied in such a way that one can actively control and intervene in the environmental dynamics. Every artist or biochemist who has knowledge about the material he or she is working with provides an example for such a behavior. Modern technologies in computer industries or biotechnologies have taken this aspect of control and manipulation of the environmental dynamics to the extreme.

It is this "constructive mode" which creates what we usually refer to as "knowledge"—be it a scientific theory or our common sense knowledge about a certain aspect of the environment. Learning, adaptation, classification, and construction are normally the processes which characterize this "standard mode" of knowledge acquisition. However, there is an alternative:

2.3 The Synthetic Mode of Knowledge Acquisition: Generating Knowledge in the Virtual Domain

From our experience we know that we can *anticipate* certain situations or events without the necessity that they happen (physically) in our environment (Riegler 2001). As has been mentioned above, prediction is a first step into that alternative mode of gaining new knowledge: By applying mental or computational operations to our knowledge, theory, model, etc., a result is anticipated and/or predicted which has not yet happened in the environment. In this sense, this result can be referred to as being "*virtual*".

Moreover, our mind, as well as computational methods (as will be shown in section 3) provide the capacity to *explore potential effects of our own (potential) actions* without having to physically externalize these actions/motor actions in our environment. For instance, whenever we are making *plans* or whenever the method of *simulation* is applied in the process of theory construction, we are entering into this new mode of knowledge acquisition. In other words, the environment is completely left aside as a source of new knowledge. The *virtual* domain becomes an alternative stage for developing new knowledge. Any kind of thought experiment or simulation experiment is an instance of this mode of knowledge acquisition. In the early days of science, when science and philosophy were still united, this was the prevalent mode of KA.

One could claim that the "constructive mode" of knowledge acquisition (as presented in section 2.2) is located in the virtual domain as well. In a way that is right: The formation of, say, a scientific theory is an operation in the representational space. However, there is always a direct feedback with the environment which is realized via the verification process of making an experiment. So, what are the new features which are introduced by this "synthetic mode" of knowledge acquisition? What is it that makes it a real alternative to the other approaches in the process of KA? There are a couple of answers to these questions:

(i) Above all, everything happens in the domain of *virtuality*, of the representational space; i.e., in this mode of knowledge acquisition, no further direct interaction with the environment (e.g., via experiments, via behavior, etc.) is necessary; empirical experiments are replaced by "*virtual experiments*"; externalization of behavior is replaced by thought experiments.

(ii) In the classical approaches, the physical environment plays the role of a *constraining* factor in theory formation and knowledge acquisition. Whatever knowledge structure functionally fits into these environmental constraints counts as adequate knowledge or theory. As the physical environment has been "lost" in the synthetic mode of knowledge acquisition, some replacement becomes necessary in order to ensure that the development of knowledge is constrained by an environment-like entity. Hence, one does not only need an operational and functional knowledge or model of the phenomenon which should be described (i.e., the result of the constructive approach to knowledge acquisition), but—above that—a *sound model of its environment*. Only if this criterion is satisfied, virtual experiments become possible: One can explore the effects of one's actions in the virtual domain. In other words, the model/knowledge of the phenomenon we are interested in is confronted with the model of the environment in a virtual experiment taking place solely in the representational/virtual domain.

Hence, the cost of detaching oneself from the physical environment is the necessity of having to develop a sound model of the environment in which the phenomenon which one is interested in can be found. Of course, this approach has a lot of implications as well as risks on the epistemological level, such as a huge problem of theory-ladenness. Despite these worries, these kinds of models have been applied with some success in many disciplines, especially in the fields of artificial life[6] or cognitive science.

(iii) The new feature which is introduced by this approach is that potential theory/ knowledge spaces can be *explored* in the virtual space of the representational system (be it in thought experiments of a cognitive system or in simulation experiments, e.g., Peschl 2001). In many cases this reduces the costs as well as the risks drastically, as the direct contact with the environment can be avoided.

(iv) The final interesting point concerns the question of how the problem of knowledge construction is approached. Whereas the classical knowledge acquisition processes follow a rather analytical approach, the mode of knowledge acquisition being discussed here stresses the aspect of *synthesis* (e.g., Braitenberg 1984). Existing theories, knowledge, theoretical entities, etc. are synthetically rearranged, taken apart, put together, tested in an virtual environment, etc. Many models in the field of artificial life (e.g., Langton 1995) are examples of this synthetic approach. Simulations are implemented in the form of a set of "ingredients" and rules for the interactions amongst them. In the course of the simulation these entities interact with each other and new structures emerge as a result of these interactions. Examples for such models are Holland's ECHO (1992) or Menczer and Belew's LEE (1994). They represent a class of so-called complex adaptive systems in which a community of distributed agents evolves in an environment with resources. However, the application of simulation environments like these as knowledge source for scientific investigation isn't without problems as one would expect and, thus, is subject of our investigations in section 4.

Now that we have discussed the abstract and conceptual issues concerning the role of virtuality in the process of KA, we are going to take a closer look at a practical application of these concepts in the context of theory development in the (natural) sciences.

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6. However, as will be discussed in section 4.1, we do not fully share the enthusiasm about artificial life.

3. Virtuality and the Development of Scientific Theories

What are the implications of these considerations for the process of science? How can the points having been made above be applied in the process of theory development? Over the last decades, an interesting shift concerning the mode of knowledge acquisition in the natural sciences can be observed. Many disciplines are moving from the classical empirical approach to extending their methods with simulation techniques. Let's take a closer look at the epistemological as well as philosophy of science implications of this shift.

3.1 The Empirical Approach in Science and Its Virtual Extension

The main task of science is to construct adequate and functionally fitting knowledge. This arises the question: What is it that makes science such a powerful tool for understanding and controlling many aspects of our world in an efficient manner? By following a particular and well established set of *methodological rules*, a particular kind of knowledge is acquired and generated which becomes the basis for the endeavor of science as well as for its applications. Any scientific activity aims at constructing possible mechanisms which could serve as explanations for an observed phenomenon. Their explanatory value consists in establishing *causal relations* between (observable) phenomena leading to such mechanisms which generate these phenomena. In the natural sciences the classical method of KA consists in the *empirical* approach as the standard means for developing a scientific theory about a certain aspect of an phenomenon.

The lower part of Figure 2 shows the classical epistemological feedback loop between the phenomenon (*explanandum*) in the environment and its theory (*explanans*). This cyclic process is based on the "epistemological tension" between a real phenomenon and its theoretical description. The goal of any scientific endeavor consists in closing this epistemological gap by applying the classical method of *conducting experiments* in which a theory or hypothesis is tested in interaction with the environmental dynamics (constraining the process of theory development; e.g., Popper 1962; Oreskes et al. 1994).

As has been shown in section 2.3, there exists an alternative method for knowledge acquisition which does not only extend the classical empirical approach but introduces a new dynamics into the whole process of theory development. Especially in those areas of science that are concerned with highly complex phenomena—such as cognition or biomolecular structures—empirical experiments are more and more extended and, in part, replaced by *simulation experiments*. In some disciplines simulation seems to be even an integral part playing the role as primary method for

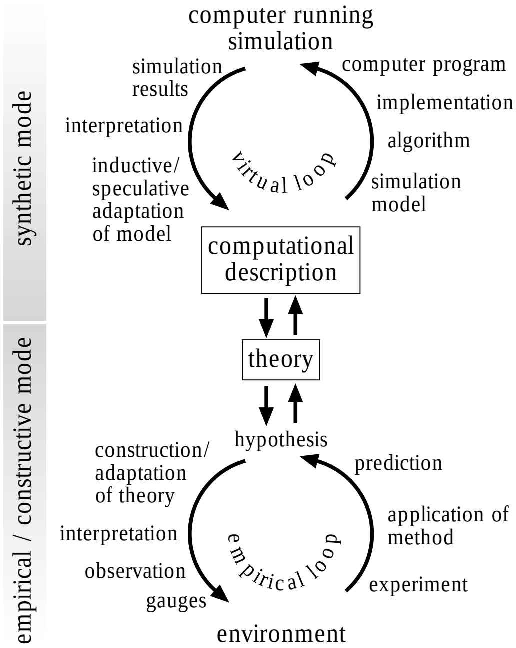

Figure 2: The process of theory construction in science. The classical feedback loop between a phenomenon in reality and its description in a theory, the "*classical empirical loop*" (lower part). The method of simulation as an extension establishing a second feedback loop for "virtual (simulation) experiments", the "virtual loop" (upper part).

knowledge acquisition (e.g., in cognitive science, artificial life). As can be seen in the upper part of Figure 2, the method of simulation introduces a *second feedback loop* which has a direct influence on the development of the particular (empirical) theory. The empirical loop is extended/ mirrored into the *domain of virtuality and computation*. The (empirically constructed) theory is transformed into a computational model and the empirical experiment is replaced by a *virtual experiment*, i.e., running a simulation of this model on a computer. The result of this virtual and cyclic simulation process is twofold: (a) It creates predictions for "real world dynamics". (b) If these predictions are not satisfactory, a possible change in the computational model may be necessary which, in turn, may suggest changes in the original (empirically based) theory. In this case, a rewritten version of the theory acts as the starting point for a new cycle of empirical and/or simulation experiments.

3.2 Methodological Steps and Epistemological Issues in Computer Simulations

In order to understand the role of simulation and its epistemological status for science, we need to analyze each (epistemologically relevant) step of abstraction involved in this process of simulation and theory construction. Let's take a closer look at the following steps and levels of abstraction which can be identified and have to be gone through in the process of theory construction (for a more detailed discussion see Peschl & Scheutz 2001).

(a) Construction of an empirical theory. Constructing a theory by applying the "empirical loop" implies a first step of *abstraction*: The phenomenon under investigation is *reduced* to a set of "relevant magnitudes" (i.e., dimensions, parameters, variables, etc.). The task of the theory is to relate these parameters to each other in such a way that predictions are possible. Furthermore, their interaction structure should provide some explanation in the form of possible mechanisms which are responsible for the generation of the observed dynamics (see also sections 2.1f).

(b) Abstract states. In this step the entities of the empirical theory (i.e., variables, operators/rules, relations, etc.) are transformed into a set of *abstract states* in which the physical conditions and constraints (e.g., time, space, etc.) of the original system become irrelevant.

(c) Purely causal and computational description. By connecting these states with each other according to the rules determined by the (empirical) theory, a *dynamical* aspect is introduced into the model. In other words, the phenomenon under investigation is reduced to some sort of *automaton* whose state transitions represent the underlying mechanism for the generation of the dynamics of the observed behavior at a highly abstract level. On this level we are dealing with *purely causal structures* and *computational processes* and it is virtually impossible to reestablish the reference to the original phenomenon under investigation.

(d) Concrete simulation models. In order to satisfy the criterion that a computational model should have a high explanatory value we have to make a step down in the hierarchy of abstraction. Only if this highly abstract automaton is transformed back into a *particular model*, these abstract causal structures are broken down into computational processes which are related/referring to concrete parameters of the original theory. Hence, by re-introducing a "meaning" to the entities of the computational/simulation model the explanatory value is increased on this level of decreased abstraction.

(e) Concrete computer programs. In the next step the model gets implemented as *particular algorithm* coded in a particular programming language.

(f) Conducting the virtual experiment. Finally, this program is *executed* on a computer—and suddenly the original (empirical) theory seems to become "alive". The fascination of such a simulation model is based on the *illusion* that the observer ascribes "real" properties, such as "being alive", to a process which is nothing but a very cleverly orchestrated change of values in variables over time (combined with a suggestive graphical output or naming of variables). The *dynamic* aspect and properties of the original theory becomes explicit in this process of running the simulation.

(g) Checking with the environment. The execution of the program (i.e., the virtual experiment) yields results which have to be compared to predictions of the theory or already existing empirical data. If there are discrepancies, adaptations in the computational model might become necessary. As a consequence, the original empirical theory might turn out to be flawed, which implies the necessity for changes in these theoret-

ical concepts. Furthermore, the results from simulation models can have an "*inspiring*" *effect* on the development of completely new or alternative conceptual perspectives and/or experimental designs.[7] In any case, a change in the empirical theory then initiates a new cycle of empirical and/or virtual experiments. These steps are repeated until the theory is such that it functionally fits into the constraints given by the dynamics of the observed phenomenon.

After having discussed these practical issues concerning virtuality and simulation in science we have to go one step further by critically questioning the (implicit) assumptions of the virtual approach to KA and—in some cases—by taking some of our claims into new realms of conceptual interpretation.

4. Virtual Science

In this section we will more closely investigate the relationship between scientific activity as knowledge acquisition process and scientific simulation. Of particular concern is the question whether *simulations* can actually *bring about new scientific knowledge*, or whether we have to disregard them as "ironic science", like other authors did (e.g., Horgan 1996); and the question whether knowledge produced by simulations permits us to understand the phenomena we are interested in, or whether it is just the attempt of a savage to gain insights by reproducing the exterior appearance of phenomena? (If the latter is true, how could mathematics ever be something else than the latter?) Finally, if we accept simulation as a valid tool for theory development, how do we set up simulation experiments and what are the mechanisms at work? In particular the problem of feature extraction will be addressed, as we want the simulation to be a simplified model of the natural phenomenon. It touches the question whether simulations can be compared to solving logical–mathematical problems. So, the basic concern of this section is the question whether simulation can be an information source for science. In particular, we will try to validate the computational approach in science.

4.1 Simulation and "Virtual Methods" as a Tool for Gaining and Generating Knowledge

It has been argued that science relies upon three different ways of gathering knowledge[8] (Jackson 1995, 1996): (1) Empirical observations; (2) Mathematical models; and (3) Computational explorations. Physical observations refer to the process of

7. Churchland et al. (1992) and Gazzaniga (2000) give examples in which simulation models suggested alternative concepts of how a cognitive phenomenon can be understood and investigated.

gathering data in order to build up an internal model. They are not a model themselves and thus are not a source of information. Observations without a model do not make sense (cf. theory-ladenness). Rather, they are necessary for a model that fits these "facts".

In this conceptual framework, only physical models have an exclusive option on discovering "reality". And only through a formal mathematical approach we can establish scientific models. Computation (and simulation) may be another source but it plays the role of a *scout* who explores the unknown before civilization (i.e., mathematics and physics) dares moving in to this area.

So one may assume that in philosophy of science these sources are often considered fundamentally different.[9] However, we consider this distinction more of an obstacle than helpful because computational models are just as good as mathematical models. Any formal logical–mathematical model can be fully mapped onto a computational system. This equivalence is based on the concepts developed by Turing (1936). Both, the mathematical and the computational approach are capable of serving as a model. The only difference is that they are based on different axioms, that they use different notations and therefore different deductive mechanisms. But despite this fundamental equivalence, computational models are *not* fully accepted as information sources, i.e., as sources that potentially increase scientific knowledge in the same way empirical observations and experiences do.

What is it that makes a model an information source? Critics of the computational philosophy of science movement disqualify such models as fancy calculators (Glymour 1993). Horgan (1996) even calls such approaches "ironic science" having no practical use. Either both mathematical and computational models are valid instruments for science or neither of them. So, why does simulation have a rather bad image? Can we rescue computational simulation as a "true" form of scientific investigation? Let us explore this question by looking at the following difference: (1) one understands what a mathematical equation, say,

$$t' = \frac{t}{\sqrt{1 - v^2/c^2}}$$

8. This is a slightly different categorization than made in section 2, because the focus is on different issues in both cases. For the sake of simplicity, we don't make a distinction between knowledge, information, and data here. They are all supposed to be synonymous for "experience" made by the scientist. Of course, such a unified perspective cannot be maintained for philosophical reasons.
9. The empirical way of gathering knowledge is aligned with the Aristotelian perspective, hence it is rooted in the environment; the mathematical–logical approach is compared to a Platonic world-view in which appearances are but imperfect instantiations of perfect ideas; hence, the idea has a primacy over the appearances of the environment.

means vs. (2) one understands what the dynamical behavior of pixels on a screen in a computer simulation means.

The discrepancy becomes clear when one takes a closer look at artificial life models such as Epstein & Axtell (1996) or Deneubourg et al. (1991). There, some pixels on the computer screen are supposed to represent ants (or people) which are engaged in some collective work. As these pixels neither include any aspect of the comparatively complex metabolism of ants nor resemble their appearance (the least problem), such simulations necessarily leave the spectator with the impression that they are deliberately designed by the programmer and steered by the program rather than having some degree of autonomy—at least this is what we would expect from "natural" systems. Therefore the entire simulation is more like a computer game than a scientific investigation. The simulated creature reacts according to a priori specified rules rather than behavioral patterns which are the result of phylogeny and ontogeny. Unlike their artificial counterparts, natural ants—at least at the collective level—seem to *understand* their environment rather than copying their behavioral repertoire from biological textbooks like programmers do. Hence we have to pose the question: Is understanding necessary? Of course, we do not hold that the capacity to understand is equal in ants and humans. The point is that successful application of virtual simulations in science necessarily involves a form of coupling between agent and system rather than mechanical–algorithmic reproduction of some recipes. This form of understanding will be investigated in the following section.

4.2 Understanding and the Reminiscence Problem: Models and "Reality"

The objection of computational simulation by several authors leads directly to a main philosophical question: What is 'correct understanding'? Richard Feynman (1985) came up with an intriguing analogy by introducing the notion of 'Cargo Cult Science'. Inhabitants of a fictive island in the South Sea had witnessed the support of goods by airplanes during World War II. Of course they would have liked this to happen again. So they started to create runways with fires along their sides; they set up a wooden hut for a man to sit in, with two wooden plates on his head as headphones, and bars of bamboo sticks looking like antennas. The *form* was perfect; everything looked the way it had been looking before. But, not surprising to us, it didn't work; no plane ever landed. From the perspective of embodiment, the lack of understanding results from a lack of being embodied in the world of Western science and technology. Isn't this like mistaking computer pixels for natural ants?

For us, who know about the functional relationships between radio headphones and antennae, this setup of the island people seems ridiculous. But isn't that a virtual reality as much as our technologically highly powered virtual worlds? Isn't a simulation just as bad as the functionally worthless wooden equipment in the above example? In this regard, scientists and philosophers speak of what has been referred to as the "reminiscence problem".

Horgan (1995) quotes Jack Cowan, according to whom "chaoplexologists" suffer from the reminiscence syndrome: "They say, 'Look, isn't this reminiscent of a biological or physical phenomenon!' They jump in right away as if it's a decent model for the phenomenon, and usually of course it's just got some accidental features that make it look like something." (p74)

This syndrome resembles the old philosophical conundrum of how to know that a model of a natural system and the system itself bear any relation to each other (see also section 4.4). How can a deductively working system, such as mathematics, allow for building bridges and flying to the moon?

Firstly, we have to recognize that between understanding a mathematical formula and pixels on a computer screen, there is no difference. It is just a matter of convention. People who cannot read and write might actually find the representational form of pixels more attractive than the mathematical equations. The use of symbols heavily relies on agreement among the members of the society which makes use of them. For outsiders they are as much a riddle as the Chinese characters (Searle 1980). Being a mathematician, or at least familiar with mathematical notation, it is easy to understand the working of the equation example (1) in the previous section by means of understanding the relationships expressed by the mathematical operators. The "–" corresponds to the instruction to take something away from something else. A computer simulation (example 2) does nothing else than establishing such deterministic relationships among computational entities—just by using a *different* set of conventionally agreed symbols and instructions.

Let's investigate the alleged relationship between a model in the virtual domain and the "original" system in more detail. What we actually do by building a model is to install a *second* source of information, namely the model itself in the following sense (Riegler 1998). Originally, we wanted to investigate the empirically observed system, but due to its complexity and/or hidden features we are neither able to sufficiently explain or understand the history of its behavior nor to anticipate the future behavior. Thus, we build a simplified analogy that we hope exhibits similar or identical behaviors.[10] In order to gain maximum security, we apply our set of scientific deductive methods[11]. This means, starting from a simplified model we apply deductions in order to reach some (desired) goal state (cf. the characterization of problem-solving in terms of means-end analysis by Newell & Simon 1963). If we, indeed, manage to arrive at our goal state, we can now translate these deductive steps

10. See also section 4.4 for a discussion of iso- and homomorphism.

back into actions in the natural system in order to achieve the goal state there as well. This parallel between model and modeled system is referred to as the Tower Bridge model (Cummins 1989; cf. also Born's LIR model in this book). And indeed, mathematics seems to validate the applicability of this strategy. We can calculate the stability of buildings before their construction takes place. Having this success in mind, we are inclined to believe that all forms of simulation work with logical deductions. The following section will address this belief.

4.3 Is Simulation Logic?

By definition, computational models are implemented on a computer which itself is a physical instantiation of a strictly logical structure, the Turing Machine. Hence, what is of primary interest is the question of whether we can equate simulation with logic.

Dennett's well-known robot analogy (1984) illustrates the shortcomings of the assumption that creatures can tackle their struggle for life in terms of ahistorical logical reasoning. A robot learns that its spare battery, its precious energy supply, is locked in a room with a time bomb set to go off soon. To solve this problem, the robot has to develop plans in order to rescue the battery which is located on a wagon. Equipped with a logical inference system, it is able to quickly reason that pulling the wagon out of the room will also move the battery out of the supposedly dangerous room. But the robot fails because it does not pay attention to the implications of its planned actions. It did not take into consideration that the bomb is also located on the wagon and, therefore, stayed close to the battery regardless of where the robot moves the wagon. A descendent of the robot is constructed in such a way that would allow it to *foresee the effects of its actions*. Taking possible side effects into account, however, does not help either. As the world is very complex, an exhaustive list of all side effects would take too long to take any action in real-time. Hence, the robot must know how to distinguish between *relevant* and *irrelevant* (side) effects. But even this process of discrimination needs an enormous amount of computation; all the more as each of the possible effects must be assigned with some (quantitative) credit in order to evaluate their usefulness. Therefore, in a logical framework, too many logical implications of even the simplest actions have to be taken into account. This results in endless computations that prevent creatures from taking those actions in an acceptable amount of time.

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11. Of course, this is only relative security, as Popper (1962) already pointed out several decades ago. He argued against the idea that the inductive principle of verification could ever lead to secure knowledge. He was, however, not aware that his falsification imperative cannot yield a secure knowledge either. One can never be sure whether he or she actually included all explanatory components that show that a theory is definitely wrong. Cf. the "imaginary case of planetary misbehavior" in Lakatos (1970) which shows that even falsification is impossible due to the infinite repertoire of possible new auxiliary hypotheses that rescues the theory at stake.

Not only (artificial and biological) animals suffer from this problem. Let us consider an example from the history of science. Assume, for a moment, that science is about establishing theories out of empirical data, i.e., scientists stroll through Nature, observe a certain range of phenomena they are (emotionally?) attached to and produce protocol data of their perceptions. Afterwards they sit down and try to distill their data into the shortest description possible —cf. Wigner's "unreasonable effectiveness of mathematics in the natural sciences" (1960). The latter part is perfectly "simulated" by computer programs such as BACON (Langley et al. 1987). It receives data about distances and revolutions of certain planets and is asked to find out about lawful behavior of those heavenly bodies. And indeed, equipped with the right heuristics it is quite simple to come up with Kepler 1, 2, 3. The achievements of the great astronomers numerically crunched into dust? Certainly not. Kepler had first to wade through a huge bulk of data which was mostly the heritage of Tycho Brahe (Kozhamthadam 1994). And before that he spent many years collecting data himself. Then he had to make the "right" choice, namely to find out what data items were *relevant* and which were not, and which geometrical figure would represent the orbits of the planets.

These examples show that it cannot be logic alone with which we create the mental platform which is used for deriving conclusions. Simply too many logically possible states have to be taken into account. Solely curve fitting and simple number crunching cannot be the key to scientific activity, since they leave the scientist with too many options. As an implication, any attempt to capture all available data and relationships amongst items of data would require almost infinite logical–computational performance. We have to somehow reduce the variety and *impose restrictions*. In other words, the above insight forces us to focus on the question: What are *relevant features* which have to be chosen for such internal trial stages? Let us start with reviewing related model-theoretical concepts first.

4.4 What Are "Relevant Features"?

Can we find an answer to this question from a model-theoretic point of view? Here, relevant features are those that allow the model to remain deterministic with respect to the states and dynamics of the original "natural" system.[12] Deterministic means that the transition from one state to another (whether in the original system or in the model) is unambiguous: Each time you insert a coin in a drinks dispenser and press "coffee" it will provide you with coffee rather than with hot chocolate. This introduc-

12. This characterization leaves us of course with the question: What are "states" in "natural" systems? However, this is exactly the point. We cannot verify assumption concerning the allegedly "true" nature of environmental states.

es a homomorphic relationship between both systems. It means that the original system is simplified to the model system by suitably compounding its states. “Suitable compounds” refers to the fact that if there are two distinctive states A and B in the original system that both deterministically transform into another state C, there is no need to distinguish between A and B in the model system. The model will remain deterministic. The original system is linked to the model by a many–to–one transformation. Applying this transformation turns the original system into a system that is isomorphic to the model (Ashby 1957). Isomorphic means that there is a one–to–one mapping possible from the states of the one system onto the states of the other. The dispenser of the above example can be called a homomorphic model of a human waiter as it compounds the many states which are characteristic of a human into one variable, being a serving entity. If you order coffee it is irrelevant whether he or she has brown or black hair, whether he or she is short or tall.

The concept of homomorphic relations is essential to modeling. It requires subsuming a variety of features and states in the original system into a new single feature or state without losing the ability to discern among the interaction these states can undergo. If an animal is not able to distinguish between predator and prey, it might not survive. However, if it cannot tell apart a lion from a tiger, this will have no further consequences as it should avoid both anyway. Being able to distinguish between those two predators is superfluous knowledge with respect to the essential, i.e., relevant, interactions between animal and predator.

Unfortunately, to determine all relevant factors is far from being a feasible enterprise. Two major obstacles can be identified. Firstly, as Dennett’s example of the logically working robot demonstrates, it is not even necessary to refer to Gödel’s Incompleteness Theorem to find scientific reasoning restricted within the vast complexity of combinatorics. It is appropriate to state that from an epistemological point of view that such a situation is highly unsatisfying. On the contrary, we—like the robot in Dennett’s example—cannot spend almost endless time on building science by only taking into consideration all possible (borderline) cases. As the predator–prey example suggests, animals with far less computational power are doing much better in telling relevant features from non-relevant ones. This ability is due to their *embeddedness* in their environment (Riegler, submitted). It means that animals are equipped with phylogenetically evolved biases which determine where to draw distinctions and where not in order to avoid the full combinatorial explosion of possible associations between single features and states. The importance of embeddedness will be elaborated later in greater detail.

Secondly, note that when establishing this homomorphic relationship between waiter and drinks dispenser neither of these systems is an “original natural” system in an ontological sense.[13] It is this matter of fact to which the statement in section 4.2 is referring to: “what we actually do by building a model is to install a *second* source of information, namely the model itself". The allegedly natural system itself is already a model (in our mind). Now consider this being equally true for the relationship between mathematical equations and the piece of "reality" they are supposed to represent. However, in the realm of mathematics this seems to be a less transparent issue. The world of appearances (of reality) is at first glance totally different from the world of abstract formalisms. Therefore, any claim that *both* are the *result of the same constructively working mind* seems to be far-fetched (cf. also the paper of Diettrich in this book).

Facing these substantial philosophical problems that make it so difficult to find "relevant" features, should we consider this an impossible enterprise? It is worth looking at a broader context. So far, we have been talking about the reminiscence problem in scientific simulations, about whether logic can account for successful applications of simulations, and about iso- and homomorphism. Therefore, one might have the impression that simulation models are but vehicles for computational theory and/or science. However, we want to stress the *importance of virtuality* and *simulation* for *cognition* in general. Here, the notion of embeddedness is not restricted to creatures living in a physical environment. As argued in Riegler (submitted), only embeddedness creates the "appropriate" environment which we requested in section 2. Embeddedness means the historical mutual integration of two systems, in particular the "structural coupling" (Maturana & Varela 1980) of an organism with its environment. Only this integration brings forth the means to appropriately distinguish between relevant and non-essential aspects and, thus, is superior to a purely logical approach.

In conclusion, by stretching the definition of embeddedness to encompass any historical-dynamical enclosure of an entity within its surrounding, we can bridge the gap between mechanical simulations running on a computer, and the form of simulation that is central to cognition.

What are the implications for setting up scientific simulations? Clearly, embeddedness requires a historical component. Not paying attention to this aspect means acting like the cargo cult people who believe that merely copying the outer appearance into their simulation of an airport will yield the desired result. It is like mixing ingredients of a system in a random order and expecting that the system will work nevertheless. In general, however, systems undergo a *development* (they have an ontogeny and, in case of organic systems, also a phylogeny), i.e., a temporally unique order of putting their components together. Building a house starting with the roof is a trivial example of how wrong it is to neglect the temporal imperative.

Finally, there are philosophical consequences. That mathematical formulae seem to be applicable to problems of architecture and other basic and applied sciences has a similar reason. It has been suggested that reality is algorithmically compressible into mathematical equations (cf. Wigner 1960). What we propose in this paper follows an

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13. In other words, such a homomorphism is system-relative rather than ontologically given.

inverse logic: It is the *formal description* which *shapes the grid through* which we seek verification in empirical experiments, and which provides us with instructions for how to build artifacts. As argued above, we do not model a “real” thing; any homomorphic modeling takes always place between two entities that are already models themselves, i.e., between entities in/of our mind. This can be claimed for virtual simulation[14] experiments as much as for mathematical models. We find here a bit of the Platonic idea that objects are imperfect realizations of perfect ideas. Indeed, what we claim is not that mathematics is the compressed description of the world but that “reality” is constructed by applying (formal) descriptions. The same holds true for simulation models, as they are, as argued before, another kind of formal descriptions. A simulation that mirrors appearances of a phenomenon in the environment is a cargo cult simulation. What is required is a “*mental embeddedness*”. Only if simulation gives rise to—in a non-ontological sense—creating new structures in our environment can we consider the simulation successful. Of course, it is not only mathematics and simulations that have the potential to bring forth reality—there are many more mental tools do not have any “formal” claim—but simulations may play a vital role in tackling complex scientific problems in a systematic and powerful manner.

5. Conclusion: What is the Value of Virtuality (in Science)?

Recent developments in the natural sciences have shown that theory development is increasingly based on conducting virtual experiments. This is mainly due to the fact that the phenomena of interest have become increasingly complex. Furthermore, as an implication of this complexity, the costs of empirical experiments have risen exponentially due to the problem of finding the “relevant pathways” in the space of possible theories (cf. the discussion about identifying “relevant features” in section 4.4). One way out of this problem of lack of information is to partially replace the empirical approach by virtual experiments: i.e., the process of theory development and of testing hypotheses is transferred to the *domain of virtuality*. Modern computer technology has made it possible to imitate what our cognitive processes are doing naturally, namely to explore the space of possible theories, models, or knowledge and the effects of their possible actions on the environment without ever having to leave the realm of virtuality. The criteria for such a successful simulation model have been discussed in this paper.

Besides economical reasons concerning the process of KA in science, one of the most important contributions of the virtual approach to the field of science lies in

14. Virtual simulations are those implemented in computer programs or carried out in the mind.

increasing the explanatory value of a theory. While this is partly accomplished by introducing dynamic aspects into the theory,[15] the main reason is the following. When using virtual simulation methods for theory development, one is forced to bring the theory into an *operational form*. Such a form has an algorithmic character and is therefore more easily comprehensible by an external person. Furthermore, such a theory is less trapped in (unimportant) “microscopic” details of the phenomenon under investigation. Rather, it is situated on a more *conceptual and operational level*. This has a positive effect on the cooperation between disciplines because the structure of the virtual domain forces the participants to use the same language, otherwise they will not be able to make (virtual) experiments.

When dealing with computational models, one should keep in mind, however, that one of the main (original) reasons for transferring the process of theory development into the virtual domain (by using simulations) is a *lack of full accessibility* (in time and/or in space) to the phenomena of interest (e.g., Oreskes et al. 1994). Hence, one is obliged to be careful with making claims on the basis of such models, because their task is exactly to bridge this lack of information by replacing environmental constraints by “virtual constraints”.

In section 4 we even went one step further by proposing a new interpretation of the nature of virtual models. According to this new perspective it is misleading to claim that an organism “extracts” information from an environment and passes this “raw” primary representation over for further “processing” within the cognitive apparatus. As an implication, any model that merely mimics phenomena in a mechanical way is necessarily a “cargo cult”-simulation and, as such, not to be taken seriously. Only through spatio-temporal embedding, or “structural coupling”, a model can capture essential features that allow for successful prediction. This holds true especially for cognitive but also scientific mathematical models. What we maintain is that mathematics does not express a (compressed) description of “reality”. Rather, “reality” is brought forward in terms of cognitive constructions such as virtual cognitive models and, in particular, mathematical descriptions. From this point of view, the question of whether a virtual model is a source of information becomes trivial. However, the essential point is that it has to be embedded (“synchronized”) within other models our world-view consists of. Using bamboo sticks as replacement for antennae demonstrates the lack of embeddedness, as the native island people haven’t had any electronic engineering training.

Although such a perspective may ultimately reverse traditional conceptions, it holds a great potential to better understand the importance of virtuality.

15. I.e., by making explicit the dynamics which is in most cases already implicitly present in the empirical theory. Furthermore, sophisticated and well chosen display techniques are used for making these dynamic results “alive”.

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References

Bechtel, W. & Graham, G. (eds.) (1998) *A Companion to Cognitive Science*. Blackwell Publishers: Oxford.

Braitenberg, V. (1984) *Vehicles: Experiments in Synthetic Psychology*. MIT Press: Cambridge MA.

Brook, A. & Stainton, R. J. (2000). *Knowledge and Mind. A Philosophical Introduction*. MIT Press: Cambridge MA.

Ashby, W. R. (1957) *An Introduction to Cybernetics*. Chapman & Hall: London.

Churchland, P. S. & Sejnowski, T. J. (1992) *The Computational Brain*. MIT Press: Cambridge MA.

Cummins, R. (1989) *Meaning and Mental Representation*. MIT Press/Bradford: Cambridge MA.

Deneubourg, J.-L., Goss S., Franks N., Sendova-Franks A., Detrain C. & Chretien L. (1991) The dynamic of collective sorting robot-like ants and ant-like robots. In: J.-A. Meyer & S. Wilson (eds.) *Proceedings of the First Conference on Simulation of Adaptive Behavior (SAB)*. MIT-Press: Cambridge MA, pp. 356–365.

Dennett, D. C. (1984) Cognitive Wheels: The Frame Problem of AI. In: Hookway, C. (ed.) *Minds, Machines, and Evolution: Philosophical Studies*. London: Cambridge University Press, pp. 129–151.

Epstein, J. M. & Axtell, R. (1996) *Growing Artificial Societies. Social Science from Bottom Up*. MIT Press: Cambridge MA.

Feynman, R. (1985) *Surely You're Joking, Mr. Feynman!* New York: W. W. Norton & Company.

Foerster, H. von (1982) Molecular Ethology. An Immodest Proposal for Semantic Clarification. In: Foerster, H. von, *Observing Systems*. Intersystems Publications: Seaside, pp. 149–188. Originally published in: Ungar, G. (ed.) (1970) *Molecular Mechanisms in Memory and Learning*, Georges. Plenum Press: New York, pp. 213–248.

Gazzaniga, M.S. (ed.) (2000) *The New Cognitive Neurosciences*. MIT Press: Cambridge MA.

Glasersfeld, E. von (1984) An introduction to radical constructivism. In: Watzlawick, P. (ed.) *The Invented Reality*. New York: Norton, pp. 17–40.

Glasersfeld, E. von (1995) *Radical Constructivism: A Way of Knowing and Learning*. London: Falmer Press.

Glymour, C. (1993) Invasion of the Mind Snatchers. In: Giere, R. N. (ed) *Cognitive Models of Science*. University of Minnesota Press: Minneapolis, pp. 465–474.

Holland, J. H. (1992) *Adaptation in Natural and Artificial Systems. 2nd Edition*. MIT Press: Cambridge MA.

Horgan, J. (1995) From Complexity to Perplexity. *Scientific American* 272: 74–79.

Horgan, J. (1996) *The End of Science. Facing the Limits of Knowledge in the Twilight of the Scientific Age.* Addison-Wesley: Reading.

Jackson, E. A. (1995) No Provable Limits to 'Scientific Knowledge'. *Complexity* 1 (2): 14–17.

Jackson, E. A. (1996) The Second Metamorphosis of Science: A Second View. Working Paper 96-05-039. Santa Fe Institute: New Mexico.

Kozhamthadam, J. (1994) *The Discovery of Kepler's Laws. The Interaction of Science, Philosophy, and Religion*. University of Notre Dame Press: Notre Dame.

Lakatos, I. (1970) Falsification and the Methodology of Scientific Research Programmes. In: Lakatos, I. & Musgrave, A. (ed.) *Criticism and the Growth of Knowledge.* Cambridge University Press: London.

Langley, P., Simon, H., Bradhaw, G. L. & Zytkow, J. M. (1987) *Scientific Discovery*. MIT Press: Cambridge MA.

Langton, C. G. (ed.) (1995) *Artificial Life: An Overview*. MIT Press: Cambridge MA.

Maturana, H. R. & Varela, F. J. (1980) *Autopoiesis and Cognition: The Realization of the Living.* Reidel: Dordrecht.

Menczer, F. & Belew, R. (1994) Evolving sensors in environments of controlled complexity. In: Brooks, R. & Maes, P. (eds.) *Artificial Life IV.* MIT Press: Cambridge MA, pp. 210–221.

Newell, A. & Simon, H. (1963) GPS: A program that simulates human thought. In: Feigenbaum, E. & Feldman, J. (eds.) *Computers and Thought.* McGraw-Hill: New York, pp. 279–293.

Oreskes, N., Shrader-Frechette, K. & Belitz, K. (1994) Verification, validation, and confirmation of numerical models in the earth sciences. *Science* 263: 641–646.

Peschl, M. (2001) Constructivism, Cognition, and Science. An Investigation of Its Links and Possible Shortcomings. *Foundations of Science* 6(1–3). In press.

Peschl, M. F. & Scheutz, M. (2001) Explicating the epistemological role of simulation in the development of theories of cognition. In: *Proceedings of the 7th International Colloquium on Cognitive Science (ICCS-01)*. University of San Sebastian: San Sebastian, pp. 274–281.

Popper, K. R. (1962) *Conjectures and Refutations. The Growth of Scientific Knowledge*. Basic Books: New York.

Riegler, A. (1998) "The End of Science": Can We Overcome Cognitive Limitations? *Evolution & Cognition* 4(1): 37–50.

Riegler, A. (2001) The Role of Anticipation in Cognition. In: Dubois, D. M. (eds)

Computing Anticipatory Systems. Proceedings of the American Institute of Physics. In press.

Riegler, A. (submitted) When Is a Cognitive System Embodied? Submitted to *Cognitive Systems Research*, special issue on "Situated and Embodied Cognition", edited by Tom Ziemke.

Riegler, A., Peschl, M. & Stein, A. von (eds.) (1999) *Understanding Representation in the Cognitive Sciences*. Kluwer Academic/Plenum Publishers: New York.

Rosen, R. (1985) *Anticipatory Systems*. Pergamon Press: Oxford.

Searle, J. R. (1980) Minds, brains, and programs. *Behavioral and Brain Sciences* 1: 417–424.

Turing, A. M (1936) On Computable Numbers, with an Application to the 'Entscheidungsproblem'. *Proceedings of the London Mathematical Society*, Series 2, Volume 42, pp. 230–265.

Wigner, E. P. (1960) The Unreasonable Effectiveness of Mathematics in the Natural Sciences. *Communications on Pure and Applied Mathematics* 13: 1–14.

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